*J. theor. Biol.* (1982) **98**, 21–27

# Multiple Fröhlich Coherent States in Biological Systems: Computer Simulation

J. POKORNÝ

*Institute of Radio Engineering and Electronics, Czechoslovak Academy of Sciences, Prague* 8, 182 51, *Czechoslovakia*

(*Received* 4 *December* 1981)

Computer simulation of the energy storage in the band of normal modes with energy supply based on the kinetic equation with nonlinear (parametric) energy exchange terms indicates the possibility of the multiple Fröhlich coherent states excitation. It is mainly connected with the non-uniform transmission probabilities $\chi$ of the combined emission–absorption processes between vibration modes. The effect of the coherent states changes on the long-range interaction forces and cancer cell migration is discussed.

A band of polarization waves consisting of the normal modes is capable of the Fröhlich condensed state formation, i.e. of the coherent excitation of a single mode provided the energy supply exceeds a critical threshold value (Fröhlich, 1968*a*,*b*, 1977, 1978, 1980). It follows from the theory that the condensed state occurs in the lowest frequency mode. It is also assumed (Fröhlich, 1980) that the single coherent state may occur in a higher frequency mode if the energy losses from this mode to the heat bath are considerably smaller than those from the other modes. Computer simulation of the energy storage in the system of normal modes suggests the possibility of simultaneous existence of the coherent states in two modes with different frequencies (Pokorný, 1982). We will investigate, therefore, the conditions for the multiple condensed state excitation in the system of normal modes. We use the well-known kinetic equation (e.g. Fröhlich, 1980)

$$\dot{n}_i = s_i - \phi_i[n_i \exp(\beta\omega_i) - (1+n_i)] - \sum_j \chi_{i,j}[n_i(1+n_j)\exp(\beta\omega_i) - n_j(1+n_i)\exp(\beta\omega_j)] \quad (1)$$

where $n_i$, $s_i$ are the number of quanta and the rate of the energy supply to the mode with circular frequency $\omega_i = 2\pi f_i$, respectively; $\phi_i$, $\chi_{i,j}$ are the transition probabilities for exchange of the energy quanta; $\dot{n}_i$ is the derivative of $n_i$ with respect to time and $\beta = \hbar/kT$ has the usual meaning. We also



0022-5193/82/170021+07 $03.00 © 1982 Academic Press Inc. (London) Ltd.

use the condition for the highly non-linear energy transfer from the coherent mode. The energy in the coherent mode is assumed to be high enough to enable us to use the classical treatment as is discussed by Haken (1969). In a non-linear classical oscillator the energy is exchanged between modes with convenient frequency relations (parametric system). Simple power relations may be found in the form

$$\sum_{m,n} \frac{mP_{m,n}}{m\omega + n\omega'} = 0, \qquad \sum_{m,n} \frac{nP_{m,n}}{m\omega + n\omega'} = 0 \tag{2}$$

where $P_{m,n}$ is the power absorbed in or released from the frequency mode with circular frequency $m\omega + n\omega'$ and $m, n$ are integers. This coincides with the Manley–Rowe power relations widely used in engineering (Manley & Rowe, 1956). The right-hand sides of equation (2) depend on the non-linear oscillator symmetry and might differ from zero in a general case.

Equation (2) holds only for two basic frequencies (i.e. for $\omega, \omega'$). Similar relations for more basic frequencies may be derived. If the frequencies of some modes fulfil the combination law $\omega_i = m\omega + n\omega'$ and appropriate momentum relations are satisfied, the energy exchange takes place between these modes. For example, the energy is transferred from the lowest frequency coherent mode to the higher frequency mode (or modes), as follows from equation (2), and by the $\chi$-process back to the lower frequency modes. This leads to saturation of the energy in the coherent mode and increased pumping of the other modes considered. It follows from equation (2) that energy may be transferred even from the mode at the thermal equilibrium to other modes. For example, assuming a three mode parametric system with circular frequencies $\omega$, $\omega'$, $m\omega + n\omega'$, $n > 0, m > 0$ and $\omega$-mode coherent the energy is transferred even from $\omega'$-mode that may be at thermal equilibrium. The above treatment of the system of normal modes involves neither the anharmonic change of the energy quanta nor the possibility of the soliton-like solution nor dissociation for extremely high energy supply.

The transition probabilities $\phi_i$ and $\chi_{i,j}$ may acquire various values depending on $i$ and $j$. It reflects the different interaction terms of particular modes between the electric vibrations and between the vibrations and the heat bath. The explicit interpretation of the transition probabilities $\phi_i$ and $\chi_{i,j}$ may involve e.g. coulomb, dipole–dipole, deformation potential or polarization field–elastic field interactions. The temperature and non-linear dependencies of these quantities should be considered, too. More detailed analysis with experimental verification is necessary which is out of the scope of this contribution. Even the frequencies of the normal modes and the energy supply should be determined from the experiments.

We adopt a computer simulation model with a small number of modes (of the order of ten) in the frequency range from 0.1 to about 4 THz similar to that used in Pokorný (1982). The computer model stems from equations (1) and (2). The non-linear energy exchange given by equation (2) starts to operate if the coherent state number of quanta exceeds a prescribed value $n_l$. A desk-top computer HP 9845 B is used.

For uniform $\phi_i$ and $\chi_{i,j}$ values, respectively, the single lowest frequency coherent mode excitation is reached for the energy supply above the critical one and the highest condensed $n_1$ value is determined by $n_l$, i.e. $n_1$ depends on the assumed non-linear region (Fig. 1). It follows from our computer results that for lower $\phi_i$ values the saturated condensed state is reached at the lower energy supply (as is expected) and that the energy in excess above the thermal equilibrium values of the higher frequency modes is increased by the decrease of $\chi_{i,j}$. In the modes where $\phi_i = 0$ the excess energy above the thermal equilibrium values may be raised especially in the modes with the frequency just above the lowest one. For higher values

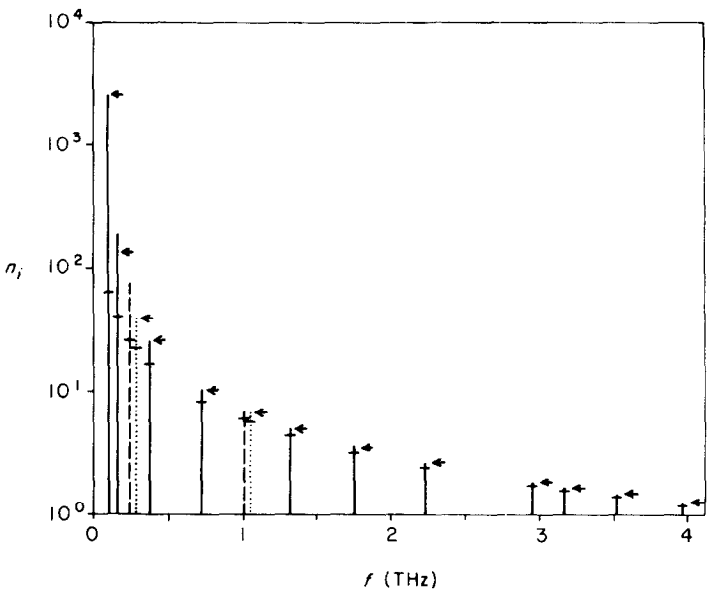



FIG. 1. Number of energy quanta $n_i$ of the normal vibration modes with different frequencies for the uniform energy supply; $\varepsilon = 10\hbar\omega_1$, $n_l = 2500$ and $\chi_{i,j} = 0{\cdot}01$ for all $i, j$. (a) $\phi_i = 0{\cdot}1$, $f = 0{\cdot}1$ THz, $f' = 1{\cdot}05$ THz, $mf + nf' = 2{\cdot}95$ THz; the $n_i$ values for $\phi_2 = 0$ or $\phi_1 = \phi_2 = 0$ and all the other parameters unchanged are not discernible from those for uniform $\phi_i$ values. (b) $\phi_1 = \phi_2 = 0$, $\phi_i = 0{\cdot}1$ for $i \neq 1$ or 2, $f = 0{\cdot}1$ THz, $f' = 0.16$ THz, $mf + nf' = 0.24$ THz. Horizontal bars indicate the thermal equilibrium values, the arrows case (a), and the upper ends of vertical lines case (b). The modes with different frequencies for (a) and (b) are plotted in dotted lines and dashed lines, respectively.

of $\chi_{i,j}$ the increase may be insignificant as in Fig. 1. Non-linear (parametric) energy transfer to these lower frequency modes causes a well-pronounced increase of the stored energy (Fig. 1). Non-uniform $\chi_{i,j}$ values have a decisive effect on the multiple coherent state excitation as follows from Figures 2, 3 and 4. If the energy supply decreases, the energy stored in the higher frequency coherent modes fades out before any change of the lowest frequency coherent mode energy occurs (Figs 2, 3 and 4).

The higher frequency coherent modes may have a multipole character and, therefore, sustain only a short-range interaction. Their significance may be limited only to small regions, e.g. to the region of cell nuclei in the biological system. Even in these cases their coherence may be supported or mediated by the lowest frequency coherent mode. The lower frequency coherent modes are assumed to have a dipole character. The coherent dipole vibrations are responsible for the long-range frequency selective interaction forces (Fröhlich, 1972, 1977, 1980) e.g. between cells (Pokorný, 1980).

The deformation connected with the excitation (e.g. the change of the energy or/and frequency) of the coherent modes may be the starting point

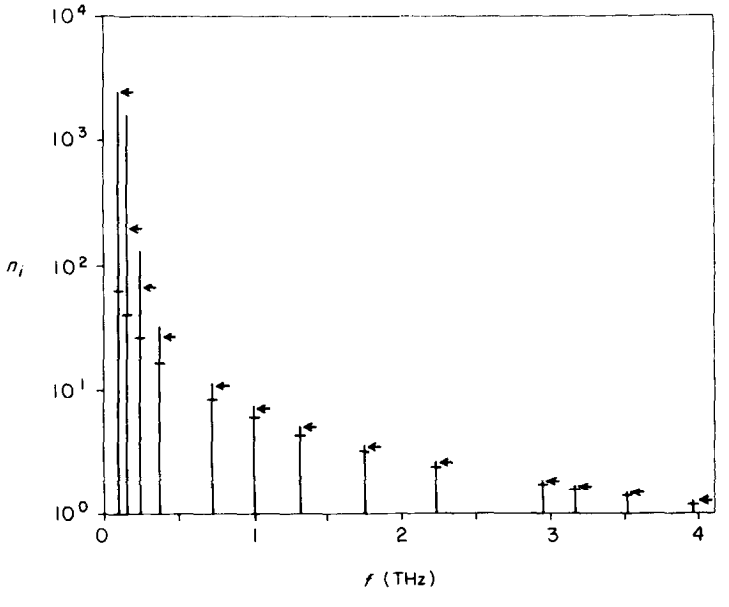



FIG. 2. Number of energy quanta $n_i$ of the vibration modes for $\varepsilon = 10\hbar\omega_1$ (the upper ends of vertical lines) and $\varepsilon = \hbar\omega_1$ (the arrows). $\phi_1 = \phi_2 = 0$, $\phi_i = 0{\cdot}1$ for $i \neq 1$ or 2, $\chi_{i,j} = \chi_{j,i} = 0{\cdot}01$ for $i < 2$, $j < 2$ and $i \geq 2$, $j \geq 2$, $\chi_{i,j} = \chi_{j,i} = 0{\cdot}001$ for $i < 2$, $j \geq 2$ and $i \geq 2$, $j < 2$, $f = 0{\cdot}1$ THz, $f' = 0{\cdot}16$ THz, $mf + nf' = 0{\cdot}24$ THz.

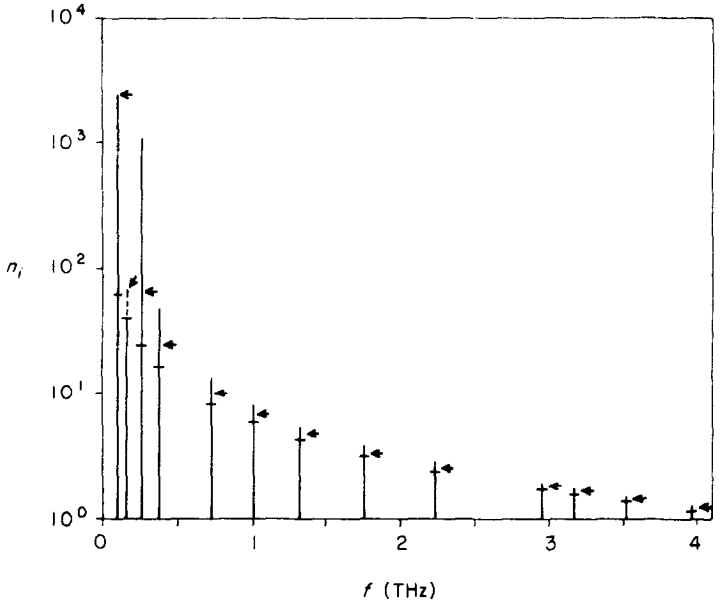



FIG. 3. Number of energy quanta $n_i$ of the vibration modes for $\phi_1 = \phi_3 = 0$, $\phi_i = 0{\cdot}1$ for $i \neq 1$ or 3, $f = 0{\cdot}1$ THz, $f' = 0{\cdot}16$ THz, $mf + nf' = 0{\cdot}26$ THz and all the other parameters identical with those in Fig. 2.

for cell division as was suggested by Fröhlich (1968*b*). A theory explaining the connection between coherent vibrations and bonds is given by Pokorný (1981). Depending on the coherent mode frequencies, energy supply, energy exchange and frequency splitting caused by the interaction, the frequency selective forces acting between cells may be attractive or repulsive. This possibility is presented for elementary cases by Pokorný (1982). It is tempting to assume, for example, that the frequency change of a coherently excited mode connected with the change of a multipole character to a dipole character of vibrations may have a well-pronounced effect on the transitions from attractive to repulsive forces. Extinction of coherent modes has to decrease or fully destroy the interaction forces. There is a wide variety of possibilities of coherent mode patterns and their changes connected with interaction forces and appropriate ones may be selected on the experimental basis. At present only qualitative experimental evidence of these forces exists (Roy, Braden & Pohl, 1981; Rowlands *et al.*, 1981).

The changes of the interaction forces may have a close relation to the migration of cancer cells at some stage of their development and their

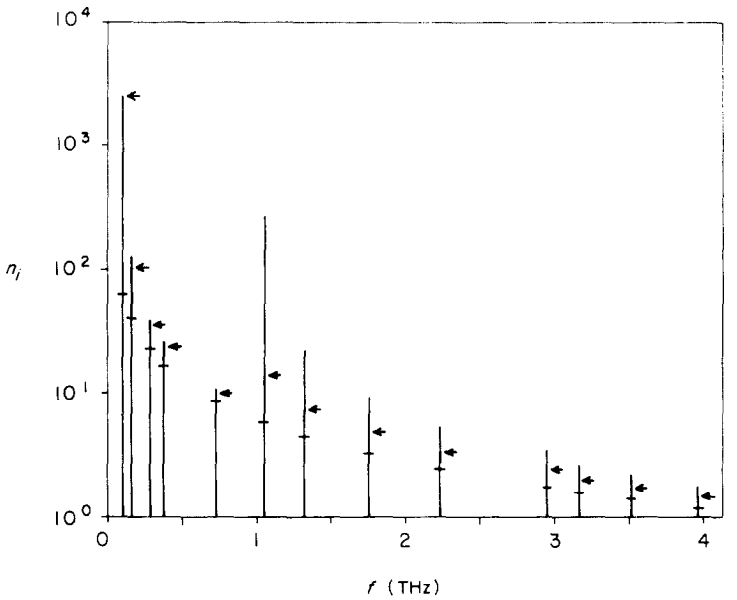



FIG. 4. Number of energy quanta $n_i$ of the vibration modes for $\phi_1 = \phi_6 = 0$, $\phi_i = 0{\cdot}1$ for $i \neq 1$ or 6, $\chi_{i,j} = \chi_{j,i} = 0{\cdot}01$ for $i < 6, j < 6$ and $i \geq 6, j \geq 6, \chi_{i,j} = \chi_{j,i} = 0{\cdot}000\,03$ for $i < 6, j \geq 6$ and $i \geq 6, j < 6, f = 0{\cdot}1$ THz, $f' = 1{\cdot}05$ THz, $mf + nf' = 2{\cdot}95$ THz, the energy supply $\varepsilon = 10\,\hbar\omega_1$ (the upper ends of vertical lines) and $\varepsilon = \hbar\omega_1$ (the arrows).

adherence to cells in tissues with convenient vibration mode structure. Differences between normal and malignant cells associated with cellular adhesiveness related to the surface properties are discussed by Abercrombie & Ambrose (1962) and by Rogers & Goel (1978). Connection of the coherent vibration mode changes with the cancer problem was suggested by Fröhlich (1968*b*, 1978).

## REFERENCES

ABERCROMBIE, M. & AMBROSE, E. J. (1962). *Cancer Res.* **22**, 525.
FRÖHLICH, H. (1968*a*). *Phys. Lett.* **26A**, 402.
FRÖHLICH, H. (1968*b*). *Int. J. Quantum Chem.* **2**, 641.
FRÖHLICH, H. (1972). *Phys. Lett.* **39A**, 153.
FRÖHLICH, H. (1977). *Riv. nuovo Cim.* **7**, 399.
FROHLICH, H. (1978). *I.E.E.E. Trans. MTT* **26**, 613.
FRÖHLICH, H. (1980). *Adv Electronics Electron Phys.* **53**, 85.
HAKEN, H. (1969). In *Theoretical Phys. and Biology* (M. Marois ed.), Amsterdam, London: North-Holland, p. 54.
MANLEY, J. M. & ROWE, H. F. (1956). *Proc. Inst. Radio Engrs* **44**, 904.
POKORNÝ, J. (1980). *Czech. J. Phys.* **B30**, 1339.

POKORNÝ, J. (1981). *Phys. Lett.* **85A,** 477.
POKORNÝ, J. (1982). *Czech. J. Phys.* **B32,** 928.
ROGERS, G. & GOEL, N. S. (1978). *J. theor. Biol.* **71,** 141.
ROWLANDS, S., SEWCHAND, L. S., LOVLIN, R. E., BECK, J. S. & ENNS, E. G. (1981). *Phys. Lett.* **82A,** 436.
ROY, S. C., BRADEN, T. & POHL, H. A. (1981). *Phys. Lett.* **83A,** 142.